Exercise 1: Scheme introduction

Deadline exercise 1: Wednesdays Nov. 28th, 23.59

Task 1: Introduction

What value is returned by evaluating each of the following expressions (in order)?

```
1.       2
2.       12.1234
3.       (+ 2 1)
4.       (+ 2 1.0)
5.       (+ (* 2 3) 4)
6.       (define age-of-adam 23)
7.       (define age-of-eva 24)
8.       (> age-of-adam age-of-eva)
9.       (define x 1.4142)
10.      (define y (* x x))
11.      y
12.      (+ x y)
13.      +
14.      (+)
15.      (lambda (a b) (+ (* a a) (* b b)))
16.      ((lambda (a) (+ a a)) 5)
17.      ((lambda (x) (* 2 x)) x)
18.      (define (foo arg)
              (+ arg 3))
19.      (foo)
20.      (define (fee)
              (+ x 5)
              x)
21.      (fee)
22.      (define (fi arg)
              (* 2 arg)
              (+ 3 arg))
23.      (fi)
```

Task 2: Lambda expressions

Test the following lambda expressions by applying them to different arguments. What are the results? Explain in words what they do.

1. `(lambda (x) x)`
2. `(lambda () 2)`
3. `(lambda (a b) (+ a b))`
4. `(lambda (a b) ((lambda (a) (+ b a)) (+ a 1)))`

Translate the following mathematical formulae to lambda expressions. Apply the lambda expressions on different values.

1. $\sqrt{x^2}$
2. $\frac{b*h}{2}$
3. $\sqrt{a^2+b^2}$
4. $celsius*1.8+32$

Task 3: Fahrenheit

In the previous exercise, you defined a lambda expression that converted degrees in Celsius to Fahrenheit. Create a procedure that does the opposite, i.e. converts from Fahrenheit to Celsius.

Procedure:

```
fahrenheit->celsius: number -> number
```

Example:

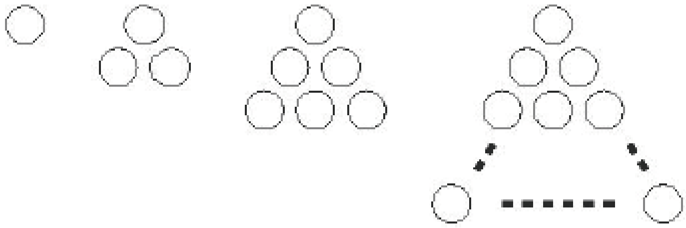
```
(fahrenheit->celsius 200)
;Value: 93.33333333333333

(fahrenheit->celsius 100)
;Value: 37.77777777777778

(fahrenheit->celsius 70)
;Value: 21.11111111111111
```

Task 4: How many pins?

In bowling one often uses ten pins positioned on four rows. How many pins are needed for five rows, six rows, or n rows (where n is a positive integer)?



1. Write a procedure that calculates the number of pins needed for n rows. The procedure should generate a recursive process. Name it `number-of-pins-rec`.

2. Same task as above, but the procedure should generate an iterative process. Name it `number-of-pins-it`.

3. Use the substitution model to show the evaluation of the following two expressions:
   a. `(number-of-pins-rec 4)`
   b. `(number-of-pins-it 4)`

Procedure:

```
number-of-pins-x: number -> number
```

Example:

```
(number-of-pins-x 1000)
;Value: 500500
```

Task 5: Exponentiation

Write a recursive procedure

```
my-expt: number x number -> number
```

that takes two arguments; a base and a number, and returns the base *b* raised to the power of the number *n*, i.e. b^n.

Using the following hint; $b^{2n} = (b^2)^n$, write a new recursive procedure called `fast-expt`.

Task 6: Testing for primality

There exist many different procedures for testing primality of numbers. One way to test if a number is a prime is to find the number's divisors. If a number *n* is prime then *n* equals the smallest integral divisor of *n*.

Implement a procedure that tests if a number is a prime based on the above method.

Another method is related to Fermat's little theorem:
If n *is a prime number and* a *is any positive integer less than* n*, then* a *raised to the* n*th power is congruent to* a *modulo* n.
Two numbers are said to be congruent modulo *n* if they both have the same remainder when divided by *n*. Trying a random number $a < n$, one can be sure that *n* is not prime if the remainder of a^n modulo *n* is not equal to *a*. However, the opposite does not always hold, i.e. a number *n* is not always prime if the remainder of a^n modulo *n* is equal to *a*. By trying more and more random $a < n$, one can get more confident that *n* is prime. This algorithm is known as the Fermat test.

Implement a Fermat test procedure.

Hints: Use primitive procedures such as *random*, *modulo* and *remainder.*
Try to abstract the different parts of the methods into primitive procedures.