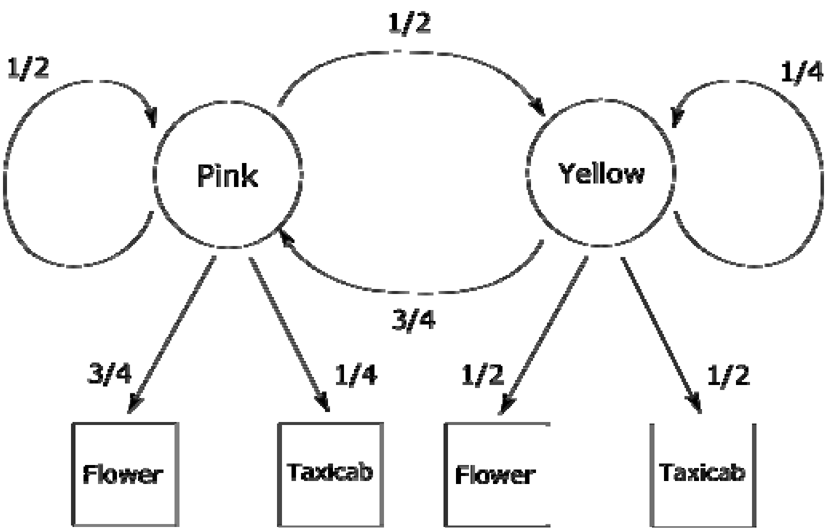
Exercise 4

Deadlines: Friday 2008.09.26 (copy) and Friday 2008.10.03 (corrected)

Consider the HMM graphically represented below that has two hidden states, Pink and Yellow and emits two symbols, Flower and Taxicab.



Task 1

Identify the parameters of the HMM.

Task 2

Calculate P({Flower, Flower, Taxicab, Flower}, {Pink, Yellow, Pink, Pink}). Assume that Pink and Yellow are equally likely to be the first states.

Remember: P(A, B) = P(A|B) P(B)

Task 3

Calculate the probability of observing the sequence {Flower, Flower, Taxicab, Flower}, i.e. P({Flower, Flower, Taxicab, Flower}) assuming that the state is equally likely to be Pink and Yellow prior to observing the sequence.

Remember:

$$P(state = k, X^1 \ldots X^i) = P(X^i \mid state = k) \sum_{\forall l} P(state = l, X^1 \ldots X^{i-1}) P(state\, l \to state\ k)$$

and note that $P(X^1 \ldots X^i) = \sum_{\forall l} P(state = l, X^1 \ldots X^i)$.

Task 4:

Determine the most probable path for the sequence {Flower, Flower, Taxicab, Flower}. Assume that the path is equally likely to start in Pink and Yellow.

Remember

The highest possible probability for all paths ending in state *k* with a prefix observation $X^1 \ldots X^i$ (denoted s_{ki}) can be calculated as

$$s_{ki} = P(X^i \mid state = k) \cdot \max_l \left[s_{l,i-1} \cdot P(state\ l \to state\ k) \right]$$

Task 5:

Calculate the probability of being in state Pink when the second Flower is emitted in the sequence {Flower, Flower, Taxicab, Flower}. How should you initialize the backward algorithm?

Remember

$$P(state\ at\ i = k \mid X^1 \ldots X^n) = \frac{P(state\ at\ i = k, X^1 \ldots X^i) P(X^{i+1} \ldots X^n \mid state\ at\ i = k)}{P(X^1 \ldots X^n)}$$